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Research Article

Robust Tracking Control for Rendezvous in Near-Circular Orbits

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This paper investigates a robust guaranteed cost tracking control problem for thrust-limited spacecraft rendezvous in near-circular orbits. Relative motion model is established based on the two-body problem with noncircularity of the target orbit described as a parameter uncertainty. A guaranteed cost tracking controller with input saturation is designed via a linear matrix inequality (LMI) method, and sufficient conditions for the existence of the robust tracking controller are derived, which is more concise and less conservative compared with the previous works. Numerical examples are provided for both time-invariant and time-variant reference signals to illustrate the effectiveness of the proposed control scheme when applied to the terminal rendezvous and other astronautic missions with scheduled states signal.

1. Introduction

Autonomous rendezvous is a key operational technology in astronautic missions that involve more than one spacecraft, such as crew exchange, spacecraft assembly, maintenance, and monitoring. Generally speaking, a complete rendezvous mission can be divided into several specific phases: launch, phasing, far range rendezvous, close range rendezvous, docking, and departure [1]. Due to some similarities between far range rendezvous and close range rendezvous, researchers would collectively refer to them as the terminal rendezvous, and as the control problem is a crucial issue for the automation of spacecraft rendezvous, it has been and continues to be an appealing area of study. In this paper, we will synthesize a robust guaranteed cost tracking controller for terminal rendezvous in near-circular orbits.

Absolute navigation and relative navigation are the two major navigation methods used in a rendezvous mission. Varying with the distance between two spacecrafts, navigation methods for each phase are varied. For the launch and phasing phases where the spacecrafts are not able to detect each other, absolute navigation is in operation, while for the last four phases, as the distance between vehicles is short enough, it is common to switch to the relative navigation mode. According to the source of the navigation data, different types of controlled plants and control methods are required. The rest of this paper mainly focuses on the rendezvous problem based on the relative navigation method, and some works relying on absolute navigation are given in [2–7].

Researchers used to establish the rendezvous model on the basis of the two-body problem. Differed by the methods used in linearizing the equations of motion, rendezvous in circular, near-circular, and elliptical orbits are the three main branches that have been studied by precursors. Among these branches, rendezvous in circular and elliptical orbits seemed more attractive. Clohessy and Wiltshire [8] established the equations for satellite rendezvous in circular orbits, which is known as Clohessy-Wiltshire equations (or Hill's equations [9]). Depending on C-W equations, Lawden put forward the primer vector theory [2], which was used by later researchers in solving the optimal N-impulse problem for rendezvous in circular orbits [10, 11]; Karr et al. [12] solved the N-impulse problem by using fuzzy control and genetic algorithms;

a rendezvous controller with obstruction avoidance was realized in [13]; rendezvous under continuous thrust was considered in [14]. Although most spacecraft rendezvous missions are expected to be accomplished in circular target orbits, the eccentricities of the target orbits are not exactly equal to zero in engineering applications, which may degrade the performance of the controllers synthesized based on the hypothesis of circular target orbits. Rendezvous problems in elliptical orbits were first raised by De Vries [15] and Tschauner [16], who extended the C-W equations to be valid for elliptical target orbits, and then Carter et al. [17, 18], Zhou et al. [19], and Karlgaard and Schaub [20] gave their solutions, respectively. However, nonlinear terms were introduced in the extended C-W equations, which complicate the control tasks. The pioneer works in resolving this conflict should be attributed to Melton [21], Anthony, and Sasaki [22], who proposed the linearized dynamic models for relative motion in near-circular orbits. But as little further work has been done, it will be meaningful for us to extend their work in this paper.

Tracking control is often employed in realizing systems that are able to track reference signals. Gao and Chen [23] synthesized a network-based H_∞ output tracking controller via a linear matrix inequality (LMI) approach. In [24], a robust H_∞ PID tracking control scheme for multivariable networked control system with polyhedral uncertainties was discussed. Zhang et al. [25, 26] studied a H_∞ step tracking control problem for discrete-time nonlinear system in a networked environment with a limited capacity, and an observer-based tracking controller for discrete-time networked predictive control systems with uncertain Markov delays was designed in [27].

In order to achieve some advanced aeronautic and astronautic tasks where trajectories and velocities are required specifically, such as obstruction and detection avoidances, tracking control is often applied on the vehicles to track the preplanned reference signals. Liao et al. [28] designed a flight tracking controller by using a LMI method. Guaranteed cost control was used in the previous works to optimize the expense on fuel and the smoothness of rendezvous trajectory. Yang and Gao [29] developed a guaranteed cost output tracking controller for the terminal phase of autonomous rendezvous. Practically, thrusts produced by actuators should be bounded. A review of anterior efforts on power-limited rendezvous and a method considering both upper and lower bounds on thrust were provided by Carter and Pardis in [30]. Via a LMI approach, Yang et al. [31] designed a controller for thrust-limited rendezvous in circular orbits. However, an integrated, concise, and less conservative manner has not been proposed yet.

Motivated by the above discussions, this paper designs a guaranteed cost tracking controller for thrust-limited rendezvous in near-circular orbits via a LMI method. Based on the works of Melton [21], we put forward a relative motion model for rendezvous in near-circular orbits, where noncircularity of the target orbit is described as a parameter uncertainty. Then the rendezvous problem is formulated into a robust tracking control problem with input saturation. According to Lyapunov stabilization theory, the guaranteed

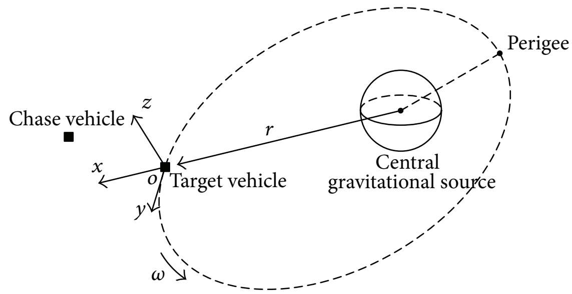


Figure 1: Cartesian coordinate system for spacecraft rendezvous.

cost tracking controller is cast into a convex optimization problem subject to LMI constraints, and the thrust saturation problem is solved by an improved, concise, and less conservative procedure. Finally, illustrative examples for both time-invariant and time-variant reference signals are presented to show the effectiveness of our proposed control scheme.

The remainder of this paper is organized as follows. Section 2 sets up the dynamic model and formulates the control problem. Section 3 gives the derivation of the robust tracking controller. Section 4 presents the numerical examples, and Section 5 draws the conclusion.

Notation. The notations used throughout the paper are given below. The superscript "T" stands for matrix transposition; $|x|$ refers to the absolute value of x; $\|\mathbf{x}\|_2$ refers to either the Euclidean vector norm or the induced matrix 2-norm. For a matrix $\mathbf{X}$, sym $(\mathbf{X})$ stands for $\mathbf{X} + \mathbf{X}^T$. For a real symmetric matrix $\mathbf{Y}$, the notation $\mathbf{Y} > \mathbf{0}$ $(\mathbf{Y} < \mathbf{0})$ is used to denote its positive- (negative-) definiteness. diag$(\cdots)$ stands for a block-diagonal matrix. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry. $\mathbf{I}$ and $\mathbf{0}$, respectively, denote the identity matrix and zero matrix with compatible dimension. If the dimensions of matrices are not explicitly stated, they are assumed to be compatible for algebraic operation.

2. Dynamic Model and Problem Formulation

In this section, a relative motion model that describes the rendezvous process in near-circular target orbits is established based on the two-body problem. Then the rendezvous control problem is converted into a robust tracking control problem, and multiple requirements on the controller are raised.

2.1. Relative Motion Model. Suppose that a target vehicle is on a near-circular orbit with a chase vehicle adjacent. It is assumed that these two spacecrafts are only influenced by a central gravitational source. To illustrate this rendezvous system, a Cartesian coordinate system is defined with the origin fixed at the centroid of the target vehicle, the x-axis aligned with the vector $\mathbf{r}$ from the Earth's center to the origin, the z-axis aligned with the angular momentum vector of the target orbit, and the y-axis satisfying the right-hand rule, as shown in Figure 1.

When a chase vehicle experiences an additional perturbing force $\mathbf{f}$, the equations of relative motions in above coordinates system can be derived in a method similar to that given in [32, 33]:

$$\ddot{x} = 2\frac{\mu}{r^3}x + \dot{\omega}y + \omega^2 x + 2\omega\dot{y} + \frac{f_x}{m}, \quad (1a)$$

$$\ddot{y} = -\frac{\mu}{r^3}y - \dot{\omega}x + \omega^2 y - 2\omega\dot{x} + \frac{f_y}{m}, \quad (1b)$$

$$\ddot{z} = -\frac{\mu}{r^3}z + \frac{f_z}{m}, \quad (1c)$$

where x, y, and z are the components of the relative position; μ is the gravitational parameter; ω is the rotational rate of the xyz frame relative to the gravitational source; f_x, f_y, and f_z are the components of the control force $\mathbf{f}$; and m is the mass of the chase vehicle.

In order to utilize linear control theory, model (1a)–(1c) should be linearized further. Generalized Lagrange's expansion theorem [34] needed by subsequent linearization procedure is introduced as a lemma here.

Lemma 1 (see [34]). *Let y be a function of x in terms of a parameter α by*

$$y = x + \alpha\phi(y). \quad (2)$$

Then, for sufficiently small α, any function $F(y)$ can be expanded as a power series in α:

$$F(y) = F(x) + \sum_{n=1}^{\infty}\frac{\alpha^n}{n!}\frac{d^{n-1}}{dx^{n-1}}\left[\phi(x)^n\frac{dF(x)}{dx}\right]. \quad (3)$$

According to the conversions between the orbit parameters and Kepler's time equation $E = M + e\sin E$, nonlinear terms in (1a)–(1c) can be rewritten as the functions of E, where E denotes the eccentric anomaly of the target vehicle, $M = n(t - t_p)$ is the mean anomaly of the target vehicle, t_p is the time of periapsis passage, and e is the eccentricity of the target orbit. When eccentricity e is sufficiently small, by Lemma 1, the nonlinear terms can be expanded as power series in constant e. Truncated at order e, we obtain the linearized results in (4a)–(4d):

$$\frac{\mu}{r^3} = n^2\left(\frac{a}{r}\right)^3 = n^2\left(\frac{1}{1 - e\cos E}\right)^3 \approx n^2(1 + 3e\cos M), \quad (4a)$$

$$\omega = \frac{h}{r^2} = n\left(\frac{1}{1 - e\cos E}\right)^2 \approx n(1 + 2e\cos M), \quad (4b)$$

$$\omega^2 = \left(\frac{h}{r^2}\right)^2 = n^2\left(\frac{1}{1 - e\cos E}\right)^4 \approx n^2(1 + 4e\cos M), \quad (4c)$$

$$\dot{\omega} = \frac{-2h}{r^3} = -2n^2\frac{e\sin E}{(1 - e\cos E)^4} \approx -2en^2\sin M, \quad (4d)$$

where $n = \sqrt{\mu/a^3}$ is the mean motion of the target vehicle; h is the angular momentum of the target orbit; r is the radius of the target vehicle; and a is the semimajor axis of the target orbit. By forming state vector $\mathbf{x}(t) = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$ and input vector $\mathbf{u}(t) = [f_x, f_y, f_z]^T$ and substituting (4a)–(4d) into (1a)–(1c), the first-order relative motion model for rendezvous in near-circular orbits can be rewritten in a matrix form as

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta\mathbf{A})\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad (5)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \frac{1}{m}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$\Delta\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 10en^2\cos M & -2en^2\sin M & 0 & 0 & 4en\cos M & 0 \\ 2en^2\sin M & en^2\cos M & 0 & -4en\cos M & 0 & 0 \\ 0 & 0 & -3en^2\cos M & 0 & 0 & 0 \end{bmatrix}, \quad (6)$$

where norm-bounded matrix $\Delta\mathbf{A}$ that contains the eccentricity of the target orbit is defined as the noncircular uncertainty. Moreover, the uncertain matrix $\Delta\mathbf{A}$ can be factorized as

$$\Delta\mathbf{A} = \mathbf{E}_1\mathbf{\Lambda}\mathbf{E}_2, \quad (7)$$

where $\mathbf{\Lambda}$ bounded by $\mathbf{\Lambda}^T\mathbf{\Lambda} < \mathbf{I}$ is a time-variant matrix with Lebesgue measurable elements; and $\mathbf{E}_1$ and $\mathbf{E}_2$ are two constant matrices with proper dimensions.

Adopting (5) as the plant model for rendezvous system, the controller can be simpler and more robust. As mentioned in Section 1, most of the rendezvous missions were conducted in near-circular orbits. So for engineering applications, the wisdom of synthesizing the rendezvous controllers on the basis of model (5) is that, comparing with the circular-orbit model, the near-circular-orbit model (5) is more precise, which can guarantee the robustness and performance of the controllers, and that, comparing with the control schemes based on the elliptical-orbit model, the controllers based on model (5) are easier to design.

2.2. Problem Formulation. Assume that reference state vector $\mathbf{x}_r$, which is expected to be tracked by the chase vehicle, is generated in real time or has been scheduled beforehand and can be either time invariant or time variant. To assess the error between state vector $\mathbf{x}(t)$ and the reference signal $\mathbf{x}_r$, tracking error $\mathbf{x}_e(t)$ is defined as

$$\mathbf{x}_e(t) = \mathbf{x}(t) - \mathbf{x}_r. \tag{8}$$

Consider the following state feedback control law:

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}_e(t), \tag{9}$$

where $\mathbf{K}$ is the state feedback gain matrix to be determined. In order to evaluate the performance of the tracking controller, a quadratic cost function at time $\tau(\geq 0)$ is defined as

$$J(\tau) = \int_\tau^\infty \left[\mathbf{x}_e^T(t)\mathbf{Q}\mathbf{x}_e(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right] dt, \tag{10}$$

where $\mathbf{Q}$, a positive symmetric matrix, is the state weighting matrix related to the smoothness of the rendezvous trajectory and convergence rate of the tracking error; $\mathbf{R}$, a positive symmetric matrix, is the control weighting matrix related to the fuel cost of the chase vehicle. Thrust constraints are also considered in our research, which can be formulated as

$$|f_i| = |u_i(t)| \leq u_{i,\max} \quad (i = x, y, z), \tag{11}$$

where $u_i(t)$ is the control thrust along the i-axis and $u_{i,\max}$ denotes the maximum control thrust that propellers can generate along the i-axis. For the purpose of dividing the input vector $\mathbf{u}(t)$ into $u_x(t)$, $u_y(t)$, and $u_z(t)$, which is helpful in designing the input saturation control law, matrices $\mathbf{U}_i$ $(i = x, y, z)$ are introduced. By defining $\mathbf{U}_x = [1, 0, 0]^T[1, 0, 0]$, $\mathbf{U}_y = [0, 1, 0]^T[0, 1, 0]$, and $\mathbf{U}_z = [0, 0, 1]^T[0, 0, 1]$, (11) can be rewritten as

$$|\mathbf{U}_i\mathbf{u}(t)| \leq u_{i,\max} \quad (i = x, y, z). \tag{12}$$

According to relative motion model (5) and restrictions on the controller described above, the rendezvous control problems to be studied can be stated as follows.

Based on controlled plant (5), design a robust tracking controller that meets the following requirements.

(i) The tracking error $\mathbf{x}_e$ converges to $\mathbf{0}$ or $\lim_{t\to\infty}\mathbf{x}(t) = \mathbf{x}_r$; in short, the chase vehicle can track a preplanned trajectory in a preplanned velocity.

(ii) The control inputs along each axis $u_i(t)$ should not exceed the given upper bounds $u_{i,\max}$ in (11).

(iii) During the rendezvous, the quadratic cost function defined in (10) should be the minimum, which means that the smoothness of the trajectory, the convergence rate of the tracking error, and the cost in fuel should be the optimal during the rendezvous.

3. Main Results

In this section, a robust guaranteed cost tracking controller with input saturation for autonomous rendezvous is designed via a LMI method, and a convex optimization problem for solving the controller is presented at the end of this section.

First of all, a lemma needed by the subsequent derivation is given. Proofs and applications for this lemma can be found in [35].

Lemma 2 (see [35]). *Given matrices $\mathbf{Y} = \mathbf{Y}^T$, $\mathbf{D}$, and $\mathbf{E}$ of appropriate dimensions,*

$$\mathbf{Y} + \mathbf{DFE} + \mathbf{E}^T\mathbf{F}^T\mathbf{D}^T < \mathbf{0}, \tag{13}$$

for all $\mathbf{F}$ satisfying $\mathbf{F}^T\mathbf{F} \leq \mathbf{I}$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\mathbf{Y} + \varepsilon\mathbf{D}\mathbf{D}^T + \varepsilon^{-1}\mathbf{E}^T\mathbf{E} < \mathbf{0}. \tag{14}$$

For each moment, the reference signal $\mathbf{x}_r$ can be treated as a constant vector. Substituting (8) and (9) into (5), the closed-loop error dynamic model can be represented as

$$\dot{\mathbf{x}}_e(t) = \dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta\mathbf{A} - \mathbf{BK})\mathbf{x}_e(t) + (\mathbf{A} + \Delta\mathbf{A})\mathbf{x}_r, \tag{15}$$

where the first term on the right-hand side plays a role in eliminating the tracking error or driving state vector $\mathbf{x}(t)$ to approach reference signal $\mathbf{x}_r$ and the second term contributes to the tracking system by maintaining the state vector around the reference signal. To track the reference state signal, the designers should determine the state feedback gain matrix $\mathbf{K}$ to ensure the closed-loop error system's (15) asymptotical stability at the point $\mathbf{x}_e(t) = \mathbf{0}$. As the reference signal has little influence on the self-stability of the error system, it is reasonable to assume it to be $\mathbf{0}$ temporarily, and then the equation of the closed-loop error system can be expressed as

$$\dot{\mathbf{x}}_e(t) = (\mathbf{A} + \Delta\mathbf{A} - \mathbf{BK})\mathbf{x}_e(t). \tag{16}$$

Equation (16) transforms the rendezvous tracking control problem into a stabilization problem. The following theorem gives the sufficient conditions for the existence of our proposed robust tracking controller.

Theorem 3. *Consider the closed-loop system* (16) *with the state feedback control law in* (9)*. For a given maximum tolerant tracking error $\mathbf{x}_{e,\max}$, if there exist a positive symmetric matrix*

$\mathbf{X}$, *a matrix* $\mathbf{Y}$ *with proper dimensions, and positive scalars* ε *and* ρ *satisfying*

$$\begin{bmatrix} \mathbf{\Psi} & \begin{bmatrix} \mathbf{X}\mathbf{E}_2^T & \mathbf{Y}^T & \mathbf{X} \end{bmatrix} \\ * & \operatorname{diag}\left(-\varepsilon\mathbf{I}, -\mathbf{R}^{-1}, -\mathbf{Q}^{-1}\right) \end{bmatrix} < \mathbf{0}, \quad (17)$$

$$\begin{bmatrix} -\rho^{-1}\mathbf{I} & \mathbf{U}_i\mathbf{Y} \\ * & -u_{i,\max}^2\mathbf{X} \end{bmatrix} < \mathbf{0}, \quad (18)$$

$$\begin{bmatrix} -\rho^{-1} & \rho^{-1}\mathbf{x}_{e,\max}^T \\ * & -\mathbf{X} \end{bmatrix} < \mathbf{0}, \quad (19)$$

where

$$\mathbf{\Psi} = \operatorname{sym}(\mathbf{A}\mathbf{X} - \mathbf{B}\mathbf{Y}) + \varepsilon\mathbf{E}_1\mathbf{E}_1^T, \quad (20)$$

then there exists a proper controller such that the closed-loop system (16) *is asymptotically stable at* $\mathbf{x}_e(t) = \mathbf{0}$, *quadratic cost* (10) *has an upper bound* ρ, *and control forces along the i-axis* ($i = x, y, z$) *are constrained below the maximum control thrust* $u_{i,\max}$.

Proof. Consider the Lyapunov function $V(t) = \mathbf{x}_e^T(t)\mathbf{P}\mathbf{x}_e(t)$, where $\mathbf{P}$ is a positive symmetric matrix. Substituting (16) into the derivative of $V(t)$, we have

$$\dot{V}(t) = \operatorname{sym}\left[\mathbf{x}_e^T(t)\,\mathbf{P}\,(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K})\,\mathbf{x}_e(t)\right]. \quad (21)$$

As is well known, the asymptotic stability of (16) can be ensured by $\dot{V}(t) < 0$.

When matrices $\mathbf{P}$, $\mathbf{K}$, $\mathbf{Q}$, and $\mathbf{R}$ are assigned appropriately, the following inequalities hold:

$$\dot{V}(t) \le -\left[\mathbf{x}_e^T(t)\,\mathbf{Q}\mathbf{x}_e(t) + \mathbf{u}^T(t)\,\mathbf{R}\mathbf{u}(t)\right] < 0. \quad (22)$$

Integrating (22) from $\tau (\ge 0)$ to ∞ and noticing that $\mathbf{x}_e(t) \to \mathbf{0}$ as $t \to \infty$, we obtain

$$J(\tau) \le V(\tau) = \mathbf{x}_e^T(\tau)\,\mathbf{P}\mathbf{x}_e(\tau) = J_{\max}(\tau), \quad (23)$$

where $V(\tau)$ also denoted as $J_{\max}(\tau)$ is defined as the upper bound of the quadratic cost at time τ. But as the quadratic cost $J(\tau)$ and its upper bound $J_{\max}(\tau)$ in (23) are acquired when reference signal $\mathbf{x}_r$ is assumed to be $\mathbf{0}$, these two parameters can only describe the system's performance in reducing the tracking error, instead of the cost in maintaining the chase vehicle at a predetermined position in a predetermined velocity when reference signal is nonzero. And as the quadratic cost in maintaining is determined by the reference signal and the time for holding, it is boundless when terminal time is unknown and may not be distinguished by different controllers. So it will be meaningless, if we take this consumption into consideration.

Consider the situation where reference signal $\mathbf{x}_r$ is time variant. The upper bound $J_{\max}$ may change as the reference signal changes, which makes the guaranteed cost controller difficult to design. To find a simple and uniform method to solve the problem, maximum tolerant tracking error $\mathbf{x}_{e,\max}$ is defined. Ensuring that (23) hold during rendezvous is one of the basic requirements for the subsequent derivation. So when reference signal is time invariant, one of a feasible assignment for $\mathbf{x}_{e,\max}$ is $\mathbf{x}_e(0)$; for the time-variant reference signal, $\mathbf{x}_{e,\max}$ should be assigned to the maximum tracking error that the users can tolerate and meets $\|\mathbf{x}_{e,\max}\|_2 \ge \max\|\mathbf{x}_e(t)\|_2$. Then the upper bound of the quadratic cost $J_{\max}$ can be expressed uniformly as

$$J_{\max} = \mathbf{x}_{e,\max}^T\mathbf{P}\mathbf{x}_{e,\max}. \quad (24)$$

Substituting (9) and (21) into inequalities (22), the inequalities can be transformed into

$$\operatorname{sym}\left[\mathbf{P}\,(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K})\right] + \mathbf{Q} + \mathbf{K}^T\mathbf{R}\mathbf{K} < \mathbf{0}. \quad (25)$$

When inequality (25) holds, it means that the error system is asymptotically stable at $\mathbf{x}_e(t) = \mathbf{0}$ and (24) is an upper bound of the quadratic cost. To avoid solving inequality (25) that contains time-variant term $\Delta\mathbf{A}$, substituting (7) into (25), there is

$$\begin{aligned} &\operatorname{sym}\left[\mathbf{P}\,(\mathbf{A} - \mathbf{B}\mathbf{K})\right] + \mathbf{Q} + \mathbf{K}^T\mathbf{R}\mathbf{K} \\ &\quad + (\mathbf{P}\mathbf{E}_1)\,\mathbf{\Lambda}\mathbf{E}_2 + \mathbf{E}_2^T\mathbf{\Lambda}^T(\mathbf{P}\mathbf{E}_1)^T < \mathbf{0}. \end{aligned} \quad (26)$$

By Lemma 2, there exists a positive scalar ε that can ensure (26) by

$$\mathbf{\Omega} + \varepsilon\,(\mathbf{P}\mathbf{E}_1)\,(\mathbf{P}\mathbf{E}_1)^T + \varepsilon^{-1}\mathbf{E}_2^T\mathbf{E}_2 < \mathbf{0}, \quad (27)$$

where

$$\mathbf{\Omega} = \operatorname{sym}\left[\mathbf{P}\,(\mathbf{A} - \mathbf{B}\mathbf{K})\right] + \mathbf{Q} + \mathbf{K}^T\mathbf{R}\mathbf{K}. \quad (28)$$

By Schur complement, inequality (27) is equivalent to

$$\begin{bmatrix} \mathbf{\Xi} & \begin{bmatrix} \mathbf{E}_2^T & \mathbf{K}^T & \mathbf{I} \end{bmatrix} \\ * & \operatorname{diag}\left(-\varepsilon\mathbf{I}, -\mathbf{R}^{-1}, -\mathbf{Q}^{-1}\right) \end{bmatrix} < \mathbf{0}, \quad (29)$$

where

$$\mathbf{\Xi} = \operatorname{sym}\left[\mathbf{P}\,(\mathbf{A} - \mathbf{B}\mathbf{K})\right] + \varepsilon\mathbf{P}\mathbf{E}_1\mathbf{E}_1^T\mathbf{P}^T. \quad (30)$$

Let $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{Y} = \mathbf{K}\mathbf{P}^{-1}$. Pre- and postmultiplying (29) by $\operatorname{diag}(\mathbf{X}, \mathbf{I})$, inequality (29) can be transformed into LMI (17). To minimize the upper bound $J_{\max}$, assume that there exists a scalar ρ that meets

$$J_{\max} = \mathbf{x}_{e,\max}^T\mathbf{X}^{-1}\mathbf{x}_{e,\max} < \rho. \quad (31)$$

By Schur complement, inequality (31) is equal to

$$\begin{bmatrix} -\rho & \mathbf{x}_{e,\max}^T \\ * & -\mathbf{X} \end{bmatrix} < \mathbf{0}. \quad (32)$$

In order to design a controller with input saturation, squaring both sides of (12) and dividing the result by $u_{i,\max}^2$, we can readily have

$$u_{i,\max}^{-2}\left[\mathbf{U}_i\mathbf{K}\mathbf{x}_e(t)\right]^T\mathbf{U}_i\mathbf{K}\mathbf{x}_e(t) \le 1. \quad (33)$$

According to (31) and the assignment rules of the maximum tolerant tracking error $\mathbf{x}_{e,\max}$, inequalities (34) hold

$$\rho^{-1}\mathbf{x}_e^T(t)\,\mathbf{P}\mathbf{x}_e(t) < \rho^{-1}\mathbf{x}_{e,\max}^T\mathbf{P}\mathbf{x}_{e,\max} < 1. \quad (34)$$

Combing (33) and (34), thrusts can be constrained below $u_{i,\max}$, when inequalities (35) hold

$$u_{i,\max}^{-2}(\mathbf{U}_i\mathbf{K})^T\mathbf{U}_i\mathbf{K} < \rho^{-1}\mathbf{P}. \quad (35)$$

From inequalities (34), we can see that $\mathbf{x}_{e,\max}$ should be assigned carefully by considering both the mobility of the chase vehicle and the tendency of the reference signal, or if the situation, $\mathbf{x}_e(t) > \mathbf{x}_{e,\max}$, happens, inequality (35) will be invalid, which may result in thrust exceeding, performance degradation, or even system instability.

Based on Schur complements, inequalities (35) can be ensured by

$$\begin{bmatrix} -\rho^{-1}\mathbf{I} & \mathbf{U}_i\mathbf{K} \\ * & -u_{i,\max}^2\mathbf{P} \end{bmatrix} < \mathbf{0}, \quad (36)$$

where $i = x, y, z$. Pre- and postmultiplying (36) by $\operatorname{diag}(\mathbf{I}, \mathbf{P}^{-1})$ and by the definitions, $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{Y} = \mathbf{K}\mathbf{P}^{-1}$, matrix inequalities (36) are equivalent to (18). When ρ^{-1} is treated as a variable, to linearize matrix inequality (32), pre- and postmultiplying it by $\operatorname{diag}(\rho^{-1}, \mathbf{I})$ yield LMI (19). This completes the proof. □

From (31) it is clear to see that the upper bound of quadratic cost $J_{\max}$ will be minimized, if we take action to minimize the positive scalar ρ. So to minimize ρ, another positive scalar w is introduced, which meets $w > \rho > 0$. By Schur complement, the optimization problem of $J_{\max}$ and ρ is converted to an optimization problem of w:

$$\begin{bmatrix} -w & 1 \\ 1 & -\rho^{-1} \end{bmatrix} < 0. \quad (37)$$

Finally, the guaranteed cost controller with input saturation for autonomous rendezvous in near-circular orbits can be obtained by solving the following convex optimization problem:

$$\begin{aligned} &\min_{\varepsilon,\rho^{-1},\mathbf{X},\mathbf{Y}} \quad w, \\ &\text{s.t.} \quad (17),\ (18),\ (19)\ \text{and}\ (37). \end{aligned} \quad (38)$$

Problem (38) can be solved by the commercial software. An optimal solution that consists of ε, ρ, w, $\mathbf{X}$, and $\mathbf{Y}$ will be obtained, and the state feedback gain matrix can be figured out by $\mathbf{K} = \mathbf{Y}\mathbf{X}^{-1}$.

In [36–41], control thrusts are limited by a preset upper bound on the performance cost, which inevitably causes the systems to be conservative. However, we improve the previous works in inequality (35) by restraining the control forces with an optimal and minimum upper bound ρ that is being optimized while solving (38), so, from Theorem 3 and optimization problem (38), a less conservative robust tracking controller with input saturation can be obtained.

4. Simulation Results and Discussion

In this section, two examples are presented to demonstrate the effectiveness and advantages of the control scheme presented above. The first example is for the situation in which the reference signal is time invariant, while the other example is for the situation where the reference signal is time variant, and both of the simulations are carried out in a two-body system.

Firstly, a simulated scene of spacecraft rendezvous is set. Consider a pair of adjacent spacecrafts. The target vehicle is circling on a low earth orbit with semimajor axis $a = 7082.253$ km and eccentricity $e = 0.05$. Thus, the mean motion of the target vehicle is $n = 1.06 \times 10^{-3}$ rad/s. The mass of the chase vehicle is 200 kg. The maximum control thrusts along the x-, y-, and z-axis are 50 N, 50 N, and 20 N.

Then, in both examples, the matrices in (7) are assigned as

$$\mathbf{E}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2e & 4e & 0 & 8e & 0 \\ 2e & 0 & 0 & 4e & 0 & 0 \\ 0 & 0 & 0 & 0 & 6e & 0 \end{bmatrix},$$

$$\mathbf{E}_2 = \begin{bmatrix} n^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & n^2 & 0 & 0 & 0 & 0 \\ 2.5n^2 & 0 & n^2 & 0 & n & 0 \\ 0 & 0.25n^2 & 0 & -n & 0 & 0 \\ 0 & 0 & n^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & n^2 \end{bmatrix},$$

$$\boldsymbol{\Lambda} = \operatorname{diag}(\sin M, -\sin M, \cos M, \cos M, -0.5\cos M, \cos M), \quad (39)$$

where the mean anomaly $M = nt$. To figure out the controller in the form of (9), we should acquire the state feedback gain matrix $\mathbf{K}$ by solving convex problem (38), such that closed-loop system (16) is asymptotically stable at $\mathbf{x}_e(t) = \mathbf{0}$ with an upper bound $J_{\max}$ of quadratic cost and input saturation. As mentioned in the introduction, the propulsive thrust discussed in these two examples can vary continuously.

4.1. Example for Time-Invariant Reference Signal. Suppose that the chase vehicle starts at a point which is 3000 m, −4000 m, and 20 m from the target vehicle along the x-, y-, and z-axis and the initial velocities of the chase vehicle are −3 m/s, 4 m/s, and −0.02 m/s along each axis. Thus, the initial state vector of the chaser is $\mathbf{x}(0) = [3000, -4000, 20, -3, 4, -0.02]^T$. In this example, although only the case where $\mathbf{x}_r = \mathbf{0}$ is discussed, the subsequent procedures are also valid for the cases where reference signals are nonzero constants. The practical situation for this example can be simply described as follows: a chase vehicle approaches a target vehicle via optimal trajectory and strategy, which is a basic problem in spacecraft rendezvous. So rendezvous accomplished when state vector equals $\mathbf{0}$.

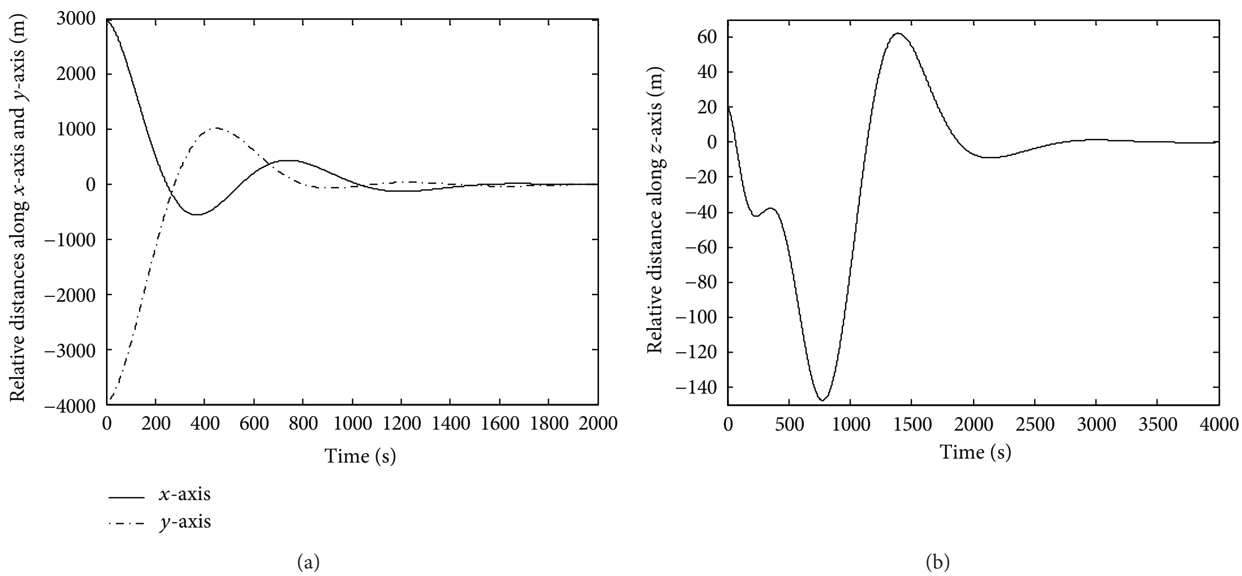


FIGURE 2: Relative distances between two spacecrafts along each axis.

Based on the guidelines for assigning the maximum tolerant tracking error, in this case, $\mathbf{x}_{e,\max}$ should be assigned to be the same as $\mathbf{x}(0)$. Solving convex optimization problem (38), we obtain (for brevity, only parts of the result are listed)

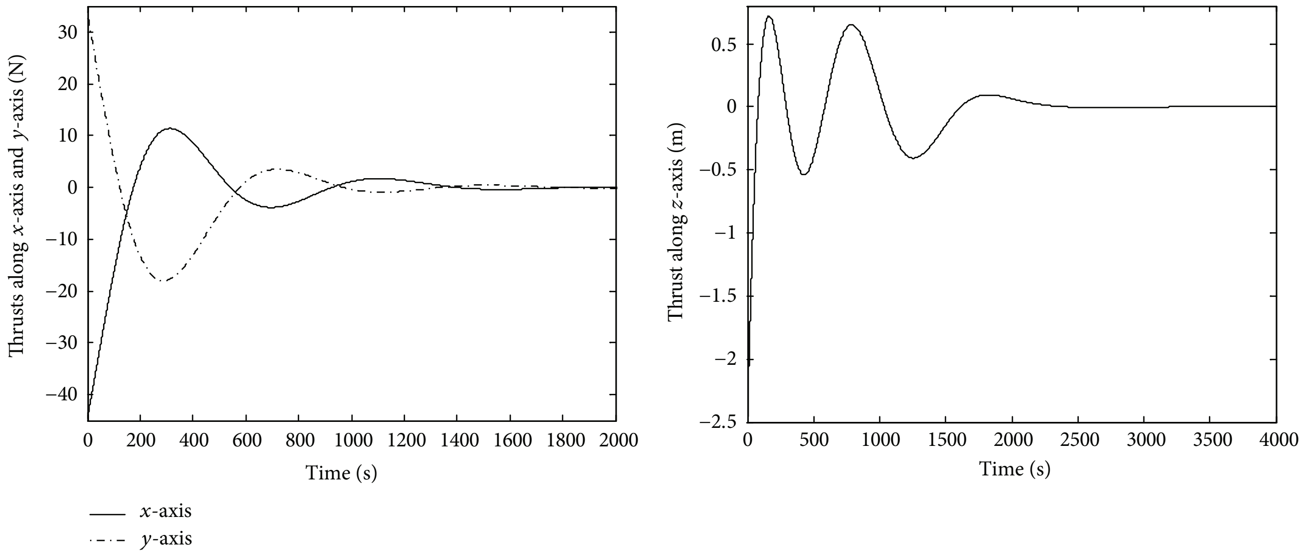
$$\mathbf{X} = \begin{bmatrix} 0.0097 & -0.0016 & 1.3846\times10^{-4} & -4.8897\times10^{-5} & 9.8620\times10^{-6} & -1.9429\times10^{-6} \\ -0.0016 & 0.0099 & 7.3825\times10^{-5} & 2.3275\times10^{-5} & -5.0499\times10^{-5} & -5.9134\times10^{-7} \\ 1.3846\times10^{-4} & 7.3825\times10^{-5} & 0.0099 & -7.8933\times10^{-7} & -4.2684\times10^{-7} & -4.8645\times10^{-5} \\ -4.8897\times10^{-5} & 2.3275\times10^{-5} & -7.8933\times10^{-7} & 7.1142\times10^{-7} & -3.9821\times10^{-7} & 1.8506\times10^{-8} \\ 9.8620\times10^{-6} & -5.0499\times10^{-5} & -4.2684\times10^{-7} & -3.9821\times10^{-7} & 6.4743\times10^{-7} & -5.8225\times10^{-11} \\ -1.9429\times10^{-6} & -5.9134\times10^{-7} & -4.8645\times10^{-5} & 1.8506\times10^{-8} & -5.8225\times10^{-11} & 4.4400\times10^{-7} \end{bmatrix},$$
$$\mathbf{Y} = \begin{bmatrix} 4.6785\times10^{-5} & -3.7093\times10^{-5} & 6.1094\times10^{-7} & 1.8544\times10^{-7} & -1.2199\times10^{-7} & 1.3959\times10^{-9} \\ -2.0179\times10^{-5} & 1.3633\times10^{-5} & -4.6829\times10^{-7} & -2.7519\times10^{-7} & 4.7088\times10^{-7} & -2.4866\times10^{-10} \\ -1.4572\times10^{-6} & -1.0233\times10^{-6} & -9.4717\times10^{-6} & 9.9580\times10^{-8} & -4.4571\times10^{-9} & 2.7637\times10^{-7} \end{bmatrix}. \quad (40)$$

Therefore, by assumptions, $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{Y} = \mathbf{K}\mathbf{P}^{-1}$, the state feedback gain matrix is

$$\mathbf{K} = \mathbf{Y}\mathbf{X}^{-1} = \begin{bmatrix} 0.0090 & -0.0053 & 4.7352\times10^{-5} & 0.9754 & -0.1368 & 3.5442\times10^{-5} \\ -0.0023 & 0.0081 & -2.0080\times10^{-5} & -0.0495 & 1.3650 & 9.9137\times10^{-7} \\ 0.0015 & 4.8836\times10^{-4} & 0.0046 & 0.3185 & 0.2075 & 1.1150 \end{bmatrix}. \quad (41)$$

With the time-invariant reference signal $\mathbf{x}_r$ and controller solved above, the relative distance between two spacecrafts during rendezvous is depicted in Figure 2. To check if the control forces acting on the chase vehicle meet requirements (ii), the control forces along each axis during the rendezvous are presented in Figure 3 and Figure 4 shows the rendezvous trajectory of the chase vehicle during the first 3500 s.

From Figures 2 and 4, it can be seen that the tracking errors or relative distances along each axis converge to zero asymptotically, though some fluctuations exist along the z-axis initially, implying that, in spite of the parameter uncertainty, the controller can stabilize the system effectively with a smooth rendezvous trajectory and low fuel cost. In Figure 3, the control forces along each axis



(a) (b)

FIGURE 3: Propulsive thrusts of the chase vehicle along each axis.

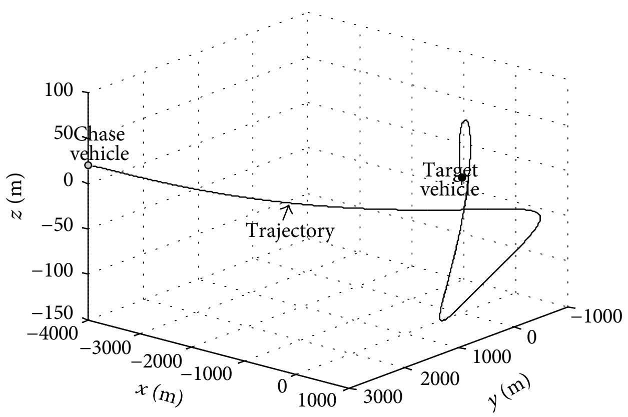


FIGURE 4: Rendezvous trajectory of the chase vehicle.

are all below the maximum control forces, indicating that the input constraints are well guaranteed by our controller. Moreover, the largest input force of three axes, −44.6226 N generated by the propulsive thruster along the x-axis at the beginning of the rendezvous, is slightly less than 50 N showing that our method is less conservative than the previous works in restraining the thrust magnitudes.

4.2. Example for Time-Variant Reference Signal. In order to study the controller's performance in tracking a time-variant reference signal, a straight-line rendezvous trajectory is planned in advance. The initial state vector is set to $\mathbf{x}(0) = [-7000, 0, 0, 0, 0, 0]$, which means that the chase vehicle starts at the position that is just below the target vehicle with no initial relative velocity. Step, ramp, and acceleration signals are applied to be the reference velocity signal along the x-axis, and the reference velocities along the y- and z-axis are assigned to zeros, which are given below

$$\dot{x}_r = \begin{cases} -0.000024t^2 + 0.024t, & 0 < t \le 500\,\text{s}, \\ 6, & 500\,\text{s} < t \le 1000\,\text{s}, \\ 6 - 0.012\,(t - 1000)\,, & 1000\,\text{s} < t \le 1500\,\text{s}, \\ 0, & 1500\,\text{s} < t \le 2000\,\text{s}. \end{cases} \quad (42)$$

From (42), we can find that the largest required acceleration along the x-axis is $0.024\,\text{m/s}^2$, which is less than $0.25\,\text{m/s}^2$, the maximum acceleration that can be generated by the chase vehicle. Referring to the initial state $\mathbf{x}(0)$ and integrating the reference velocity given in (42), we obtain the reference position signal along the x-axis as

$$x_r = \begin{cases} -0.000008t^3 + 0.012t^2 - 7000, & 0 < t \le 500\,\text{s}, \\ 6t - 8000, & 500\,\text{s} < t \le 1000\,\text{s}, \\ -0.006t^2 + 18t - 14000, & 1000\,\text{s} < t \le 1500\,\text{s}, \\ -500, & 1500\,\text{s} < t \le 2000\,\text{s}. \end{cases} \quad (43)$$

As it is designed, the chase vehicle is in the accelerated motion during the first 500 s, uniform motion during the second 500 s, uniformly decelerated motion in the third 500 s, and position holding in the last 500 s.

To demonstrate the maximum tolerant tracking error's impacts on the rendezvous system, two groups of parameters are adopted and compared. For the precise group, we set the tolerant error to $\mathbf{x}_{e,\max} = [5, 1, 1, 0.5, 0.1, 0.1]^T$. For the rough group, the tolerant error is set to

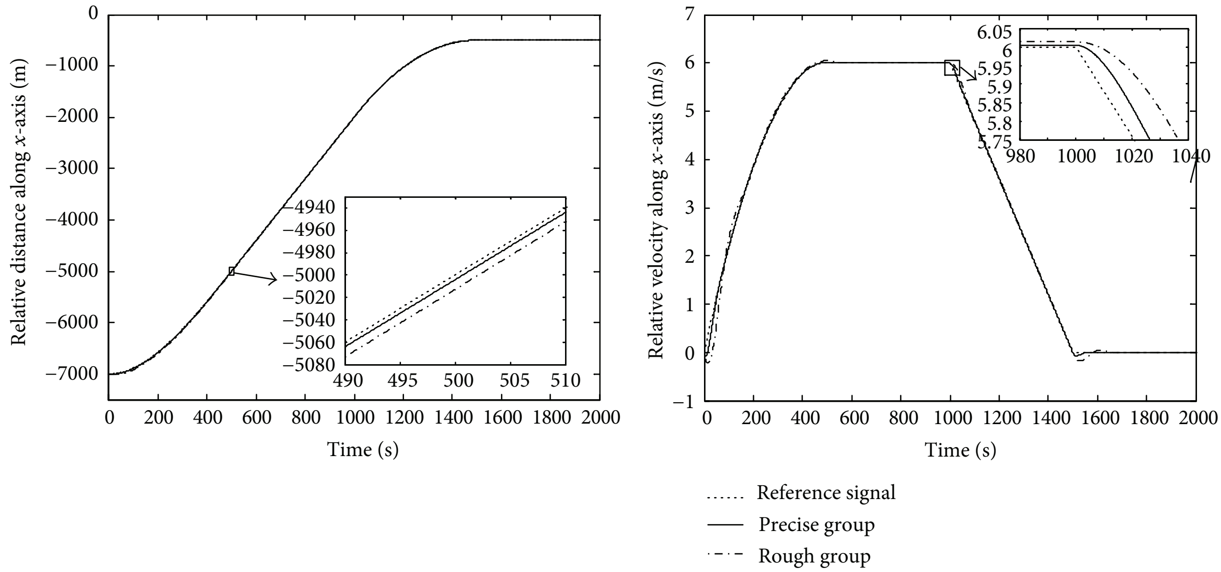


Figure 5: Relative position and velocity between two spacecrafts along the x-axis.

$\mathbf{x}_{e,\max} = [50, 1, 1, 5, 0.1, 0.1]^T$. Solving convex problem (38), for brevity, only parts of the precise group's result are listed

$$\mathbf{X} = \begin{bmatrix} 0.0984 & 3.8317\times10^{-5} & 5.3660\times10^{-4} & -0.0049 & -1.0531\times10^{-4} & -1.1207\times10^{-5} \\ 3.8317\times10^{-5} & 0.0985 & -1.2901\times10^{-5} & 1.0183\times10^{-4} & -0.0049 & 4.4791\times10^{-6} \\ 5.3660\times10^{-4} & -1.2901\times10^{-5} & 0.0967 & -9.7569\times10^{-6} & -4.3481\times10^{-6} & -0.0049 \\ -0.0049 & 1.0183\times10^{-4} & -9.7569\times10^{-6} & 4.9120\times10^{-4} & 1.4237\times10^{-7} & 2.4105\times10^{-6} \\ -1.0531\times10^{-4} & -0.0049 & -4.3481\times10^{-6} & 1.4237\times10^{-7} & 4.9256\times10^{-4} & -3.1390\times10^{-7} \\ -1.1207\times10^{-5} & 4.4791\times10^{-6} & -0.0049 & 2.4105\times10^{-6} & -3.1390\times10^{-7} & 4.8237\times10^{-4} \end{bmatrix},$$
$$\mathbf{Y} = \begin{bmatrix} 7.0369\times10^{-5} & -1.8340\times10^{-5} & -3.4114\times10^{-4} & 0.0050 & 1.1126\times10^{-6} & 5.2814\times10^{-5} \\ 6.3564\times10^{-6} & -6.3503\times10^{-7} & -3.1493\times10^{-5} & -3.1379\times10^{-7} & 0.0050 & 1.5873\times10^{-6} \\ 2.5518\times10^{-4} & 7.4186\times10^{-6} & -0.0013 & 8.4057\times10^{-6} & -3.6874\times10^{-6} & 0.0050 \end{bmatrix}. \quad (44)$$

By assumptions, $\mathbf{X} = \mathbf{P}^{-1}$ and $\mathbf{Y} = \mathbf{K}\mathbf{P}^{-1}$, the state feedback gain matrix of the precise group is

$$\mathbf{K} = \mathbf{Y}\mathbf{X}^{-1} = \begin{bmatrix} 1.0046 & -0.0216 & -0.0112 & 20.1737 & -0.0042 & -0.0813 \\ 0.0209 & 1.0002 & 0.0020 & -0.0054 & 20.1076 & 0.0281 \\ -0.0078 & 1.5830\times10^{-4} & 1.0438 & -0.1422 & 0.0150 & 20.8789 \end{bmatrix}. \quad (45)$$

With reference signals in (42), (43), and the controller solved above, the relative distance and velocity between two spacecrafts during rendezvous are illustrated in Figure 5. According to relative motion model (5), the motions along the x- and y-axis are coupling. So Figure 6 presents the control forces along these two axes of the precise group.

Figure 5 illustrates that the controller can track the step, ramp, and acceleration signals well and the maximum tolerant tracking error $\mathbf{x}_{r,\max}$ does have impacts on the performance of the controller. Generally speaking, when $\mathbf{x}_{r,\max}$ is compatible with the mobility of the chase vehicle and the tendency of the reference signal, the smaller the error is, the more precise the tracking system will be. From Figure 6, the largest control thrust of the precise group is 12.2830 N smaller than 50 N, and we can conclude that, if $\mathbf{x}_{r,\max}$ is assigned properly, the input constraint can be well guaranteed

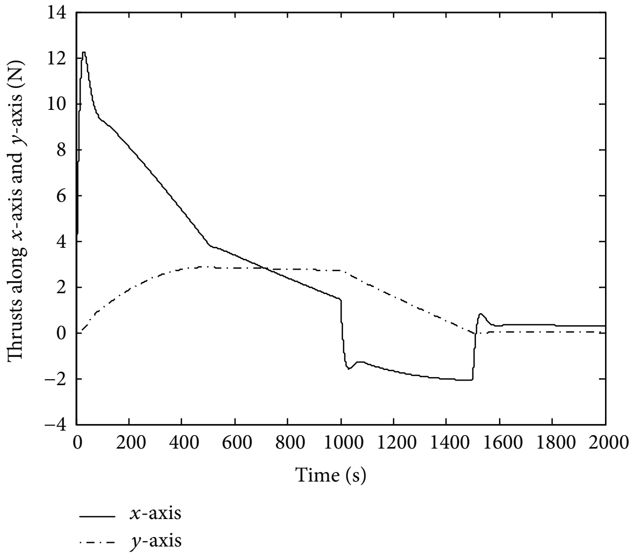


FIGURE 6: Propulsive thrusts of the chase vehicle along the x- and y-axis.

by the controller even in the situation where reference signal is time variant.

5. Conclusions

This paper has discussed a guaranteed cost tracking control problem for spacecraft rendezvous with an upper bound on thrust. A relative motion model with parameter uncertainty for rendezvous in near-circular orbits has been established. Via a LMI approach, an integrated, concise, and less conservative control scheme has been proposed. Then the controller has been demonstrated by two numerical examples with time-invariant and time-variant reference signals. The results show that our control scheme is effective for the terminal phase of rendezvous with all purposed requirements met, and, due to its strong ability in tracking reference signal, this control method should also be valid for flying by, departure, and other potential missions with planned path.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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